Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Topology

Subject Code : 4SC06TOC1 **Branch :** B.Sc.(Mathematics)

Semester : 6 **Date :** 11/05/2016 **Time :** 2:30 To 5:30 **Marks :**70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Let X = {a, b, c} and let ß = {Ø, {a}, {b}, X} be a collection of subsets of X. Is ß a topology on X? Justify your answer.

b) If U is open set of X and A is closed set of X, then show that $A \cap U^c$ is closed set of X.

c) Let T_1 and T_2 be two topologies on X. Is $T_1 \cap T_2$ a topology on X? Justify your answer with an example.

d) Let X = {a, b, c} and let ß be a co-finite topology on X. Then show that ß is a discrete topology on X.

e) Define the co-countable topology on an infinite set X.

f) Write a basis for the K-topology on R.

g) Define the closure $\bar{A}$ of a subset A of X.

h) Let A = [0, 1) be a subset of R. Find Int A and Bd A.

i) Define a continuous mapping.

j) Let I: $(X, T_1) \to (X, T_2)$ be the identity mapping. Under what condition, I is continuous?

k) Define a compact space.

l) Show that R is not compact in the usual topology.

m) Is $X = \{0\} \cup (1, 2)$ a connected subspace of R? Justify your answer.

n) Define a metric d on a non-empty set X.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If X is a countably infinite set. What can you say about comparability of the co-finite topology and the co-countable topology on X? Justify your answer. **(3)**

b) Define a topology generated by a basis B. Let B = {[a, b): a, b in R, a < b}. Then show that B is a basis for the lower-limit topology on R. **(3)**

c) Define a topology generated by a subbasis S. Show that the topology defined is, **(3)**

in fact, a topology on X.

d) Define a subspace topology T_Y on a subset Y of a topological space X. Show that the topology T_Y is, in fact, a topology on Y. **(3)**

e) Let ß be the collection of all circular regions in R^2 and Let ß" be the collection of all rectangular regions in R^2. Then show that both bases generate the same topology on R^2. **(2)**

Q-3 **Attempt all questions** **(14)**

a) Let Y = [-1, 1] be a subspace of R, and let A = {x: ½ ≤ |x| < 1}. Is A open in Y? Is A open in R? Justify your answers. **(3)**

b) Let Y be a subspace of X. If U is open in Y and Y is open in X, then show that U is open in X. Prove the same statement for closed set. **(3)**

c) If Y is a subspace of X and A is a subset of Y, then show that the topology A inherits as a subspace of Y is same as the topology it inherits as a subspace of X. **(3)**

d) Let Y be a subspace of X. Then show that A is closed in Y if and only if A = F ∩ Y, for some closed set F of X. **(4)**

e) Define an open set in a topological space X. Show that (-1, 1) is open in R. **(1)**

Q-4 **Attempt all questions** **(14)**

a) Let Y be a subspace of X. and let A be a subset of Y. Let Ā denote the closure of A in X. Then show that $\bar{A}_Y = \bar{A} \cap Y$. **(4)**

b) Let Y = (0, 1] and A = (0, ½) ⊂ Y, then show that $\bar{A}_Y = \bar{A} \cap Y$. **(3)**

c) Define a Hausdorff space X. Show that every singleton set in a Hausdorff space is closed. **(3)**

d) If X is a Hausdorff space, then show that a sequence of points of X converges to at most one point of X. **(3)**

e) Find the set of all limit points of C, where C = {1/n | n ε N}. **(1)**

Q-5 **Attempt all questions** **(14)**

a) Suppose that f: X → Y is a continuous mapping. If x is a limit point of the subset A of X, is it necessarily true that f(x) is a limit point of f(A)? Justify your answer. **(4)**

b) Let X and Y be two topological spaces, and let f: X → Y be a mapping. If for every closed set B of Y, $f^{-1}(B)$ is closed in X, then show that f is continuous. **(3)**

c) Define a homeomorphism. Let I: $(X, T_1) \rightarrow (X, T_2)$ be the identity mapping. Then show that I is a homeomorphism if and only if $T_1 = T_2$. **(3)**

d) State and prove Pasting Lemma. **(3)**

e) Let A be a subspace of X, then show that the inclusion map j: A → X is continuous. **(1)**

Q-6 **Attempt all questions** **(14)**

a) Define a basis for the metric topology induced by a metric d on X. Show that the basis defined is, in fact, a basis for the metric topology on X. **(4)**

b) Define a metric which induces the discrete topology on X. Also show that every metric space is a Hausdorff space. **(3)**

c) State and prove the sequence lemma. **(3)**

d) Let (X, d) be a metric space. Then show that $d: X \times X \rightarrow R$ is continuous. **(3)**

e) Does the standard bounded metric on R generate the usual topology on R? **(1)**

Q-7 **Attempt all questions** **(14)**

a) Define a connected topological space. Show that $Y = [-1, 0) \cup (0, 1]$ is a disconnected subset of R. **(3)**

b) Show that the image of a connected space under a continuous map is connected. **(3)**

c) Let T_1 and T_2 be two topologies on X such that T_2 is finer than T_1. What does connectedness of X in one topology imply about the connectedness in the other? Justify your answers either by proofs or counterexamples. **(3)**

d) Let X be an infinite set. Show that X is connected in the co-finite topology on X. Is the topologist's sine curve connected? Justify your answer. **(4)**

e) Define totally disconnected space, and give an example of it. **(1)**

Q-8 **Attempt all questions** **(14)**

a) Show that $X = \{0\} \cup \{1/n \mid n \in N\}$ is a compact subspace of R. Also show that (0, 1] is not compact in R. **(3)**

b) Show that every closed subspace of a compact space is compact. **(3)**

c) Let T_1 and T_2 be two topologies on X such that T_2 is finer than T_1. What does compactness of X under one topology imply about the compactness under the other? Justify your answers either by proofs or counterexamples. **(3)**

d) Define an isolated point. Show that the Cantor set is compact and uncountable. **(3)**

e) Is the topologist's sine curve compact? Justify your answer. **(2)**